The prompt is multi-part, each part being given to the students after their response to the previous part has been returned with comments.

(i) Suppose T: V→W is a linear transformation and B is a basis for V. Prove that T is an isomorphism if, and only if, T(B) is a basis for W.

(ii) Suppose T: V→W is a linear transformation and B is a basis for V. Prove that T is an isomorphism if, and only if, T(B) is a basis for W by filling in the gaps in the following structure:

> Suppose T is an isomorphism…
> ¦
> …thus we conclude T(B) is a basis.
>
> Suppose T(B) is a basis…
> ¦
> … thus we conclude T is an isomorphism.

(iii) Suppose T: V→W is a linear transformation and B is a basis for V. Prove that T is an isomorphism if, and only if, T(B) is a basis for W by filling in the gaps in the following structure:

> Suppose T is an isomorphism,
> *i.e.*, T is one-to-one and onto.
> ¦
> … thus we conclude that T(B) is a linearly independent set.
> ¦
> … thus we conclude that T(B) is a spanning set,
> and thus we conclude T(B) is a basis.
>
> Suppose T(B) is a basis,
> *i.e.*, T(B) is a linearly independent and spanning set.
> ¦
> … thus we conclude that T is one-to-one.
> ¦
> … thus we conclude that T is onto,
> and thus we conclude T is an isomorphism.

(iv) Suppose T: V→W is a linear transformation and B is a basis for V. Prove that T is an isomorphism if, and only if, T(B) is a basis for W by filling in the gaps in the following structure:

> Suppose T is an isomorphism,
> *i.e.*, T is one-to-one and onto.
> Because T is one-to-one we have that …

¦

Because B is a basis we have that …

¦

… thus we conclude that T(B) is a linearly independent set.

Because T is onto we have that …

¦

Because B is a basis we have that …

¦

… thus we conclude that T(B) is a spanning set,

and thus we conclude T(B) is a basis.